ECE 280 Spring 2008
Homework 9 Solutions

Question 1.

Using Equation 4.32:

$$S(\tau) = M_0 e^{-2\tau/T_2}$$

By maximizing the difference of the intensities using T2 of fat (T2F) and T2 of water T2W:

$$\frac{d}{d\tau}(S_{FAT}(\tau) - S_{WATER}(\tau)) = (M_0 e^{-2\tau/T_{2F}} - M_0 e^{-2\tau/T_{2W}}) = 0$$

$$(\frac{-2M_0}{T_{2F}} e^{-2\tau/T_{2F}} + \frac{2M_0}{T_{2W}} e^{-2\tau/T_{2W}}) = 0$$

$$\frac{1}{T_{2W}} e^{-2\tau/T_{2W}} = \frac{1}{T_{2F}} e^{-2\tau/T_{2F}}$$

$$\ln\left(\frac{1}{T_{2W}}\right) - \frac{2\tau}{T_{2W}} = \ln\left(\frac{1}{T_{2F}}\right) - \frac{2\tau}{T_{2F}}$$

$$\frac{2\tau}{100} - \frac{2\tau}{500} = -4.605 + 6.215$$

$$8\tau = 805$$

$$\tau = 100.625 ms$$

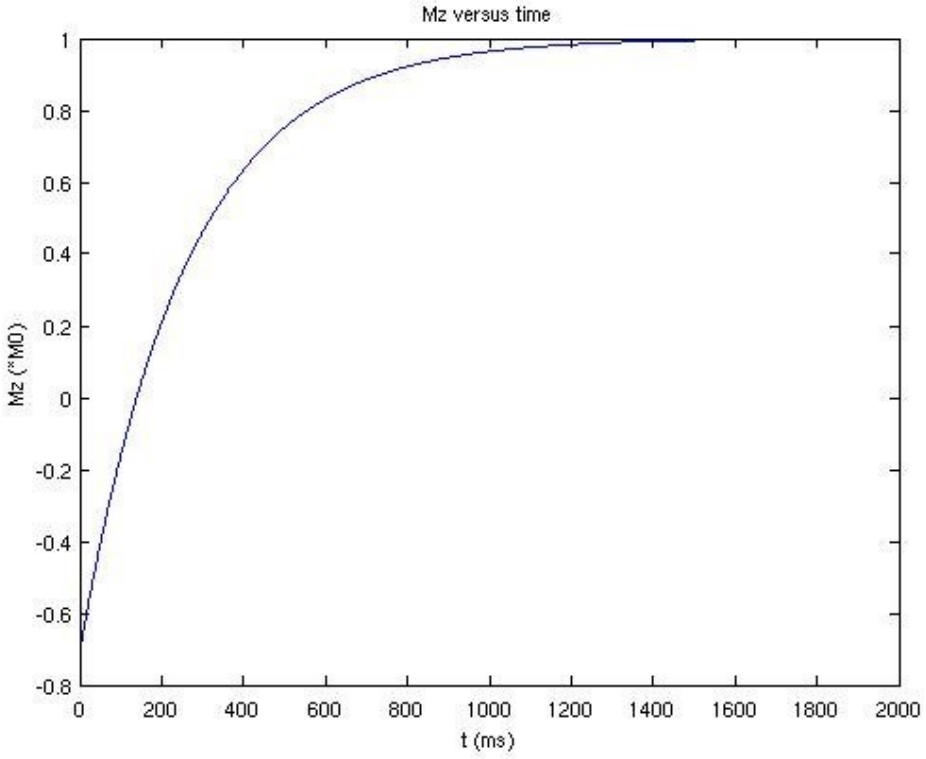
Question 2.

Using Equation 4.25:

$$M_z(t) = M_0 \cos\alpha + (M_0 - M_0 \cos\alpha)(1 - e^{-t/T_1})$$

$$M_z(t) = -M_0 + 2M_0(1 - e^{-t/T_1})$$

Using alpha = 135 degrees, we get the plot on the next page. The y-axis intersects at $-0.7M_0$.



Question 3.

The three gradients cannot be applied simultaneously because there are three steps for the image formation:

- Slice Selection
- Phase Encoding
- Frequency Encoding

Phase encoding can not be done before slice selection and frequency encoding occurs during data acquisition. Since the three steps use different gradients, they can not be applied simultaneously.

Question 4.

To get the Ernst angle, you need to maximize the SNR.

$$I(x,y) \propto \frac{\rho(x,y)(1-e^{-TR/T_1})e^{-TE/T_2^*}\sin\alpha}{1-e^{-TR/T_1}\cos\alpha}$$

Lets define:

$A = \rho(x,y)(1-e^{-TR/T_1})e^{-TE/T_2^*}$

$B = e^{-TR/T_1}$

$$I = \frac{A\sin\alpha}{1 - B\cos\alpha}$$

Maximum at dI/dα = 0, so:

$$I' = \frac{A\cos\alpha - AB\cos^2\alpha - AB\sin^2\alpha}{(1 - B\cos\alpha)^2} = 0$$

$$A\cos\alpha - AB(\cos^2\alpha + \sin^2\alpha) = 0$$

$$A\cos\alpha - AB = 0$$

$$\cos\alpha - B = 0$$

$$\alpha = \cos^{-1} B$$

$$\alpha = \cos^{-1}(e^{-TR/T_1})$$

Question 5.

The T2 value of the brain tumor is larger, so using a **T2 weighted** sequence will show a large CNR & higher intensity at the tumor.
Adding a large amount of superparamagnetic agent that accumulates in the tumor will reduce the T2 in the tumor and so **T1 weighted** sequence will be more suitable for the detection.