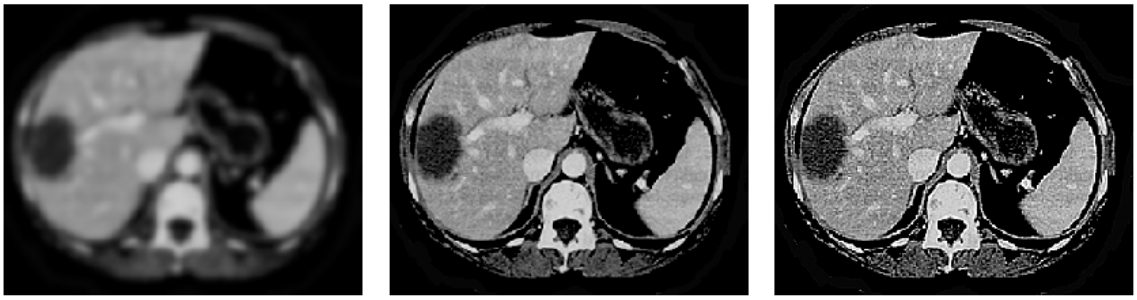
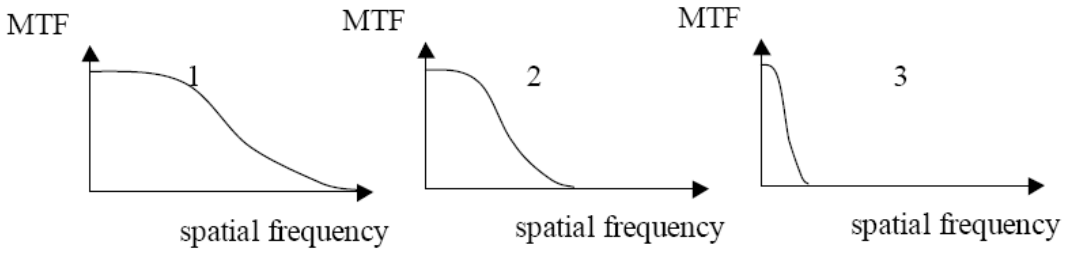
# Homework 2

Due Friday, Feb. 8- Noon, 2008
Drop Box #8 Everitt Basement

1. Shown below are three CT images: one represents the reconstructed image with no filtering, one has been high-pass filtered and the other has been low-pass filtered. For each image, A, B and C, state which form of filtering has been used, and the effects on signal-to-noise, spatial resolution, and contrast-to-noise of the filtering. Finally, match the image to one of the modulation transfer functions, 1, 2, or 3.



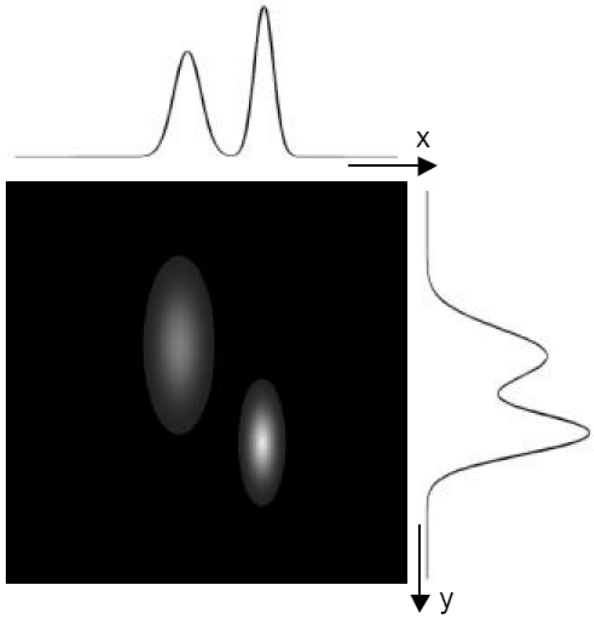
A B C




(10 points)

2.

The following image is due to two point sources. Along which axis or axes are the two points resolvable according to a) FWHM criterion and b) Rayleigh criterion (you may need to use a ruler)?




(10 points)

3.

An imaging system consists of two components with the following PSFs.

$$h_1(x) = \delta(x) \qquad \text{and} \qquad h_2(x) = e^{-2x^2}$$

a) Find the MTF of each component.

b) Find the total PSF of the system.

(10 points)

4. The average number of X-ray photons that reach the detector per unit time is $\mu$=100. Calculate the following probabilities, assuming that the Poisson statistics apply:
a) probability to detect N=1 photon per same unit time
b) probability to detect N=100 photons per same unit time
c) probability to detect N=10,000 photons per same unit time.
If the signal in 3 different measurements consists of N=1, 100, and 10,000 photons, what is the SNR in each case?

(10 points)

5. Two successive images have been recorded using a biomedical imaging device and the utility of averaging the two measurements is investigated. We know that the signal in both measurements is the same, but the noise is different. The image is sampled on each direction by 128 points. The Excel file attached contains the data sets associated with these measurements, as follows:
Column A- ideal signal, X
Column B- noise of measurement 1, $\xi_1$
Column C- noise of measurement 2, $\xi_2$

a) Plot on the same graph the following (3 graphs):
   i. Ideal signal, X, noise signal 1, $\xi_1$, measured signal 1, $\widehat{X_1} = X + \xi_1$, and measured signal 2, $\widehat{X_2} = X + \xi_2$
   ii. $\xi_1$, $\xi_2$, and $<\xi>=(\xi_1+\xi_2)/2$
   iii. $\widehat{X_1}$, $\widehat{X_2}$, and $<\widehat{X}>=\left(\widehat{X}_1+\widehat{X_2}\right)/2$

b) Compute the following
   i. Variance of $\xi_1$, variance of $\xi_2$, variance of $<\xi>=(\xi_1+\xi_2)/2$
   ii. Standard deviation of $\xi_+ = \xi_1 + \xi_2$ and $\xi_- = \xi_1 - \xi_2$
   iii. Standard deviation of $\xi_1$, $\xi_2$ and of $<\xi>=(\xi_1+\xi_2)/2$
   iv. The SNR improvement due to averaging.

(60 points)

6. Bonus problem. Using numerical Fourier transformation, outline a procedure to filter the noise out of the measurement $\widehat{X_1} = X + \xi_1$ from Problem 5 (use, for instance, Matlab). Support your results by plotting the data at each step of your procedure.

(10 points + pride)